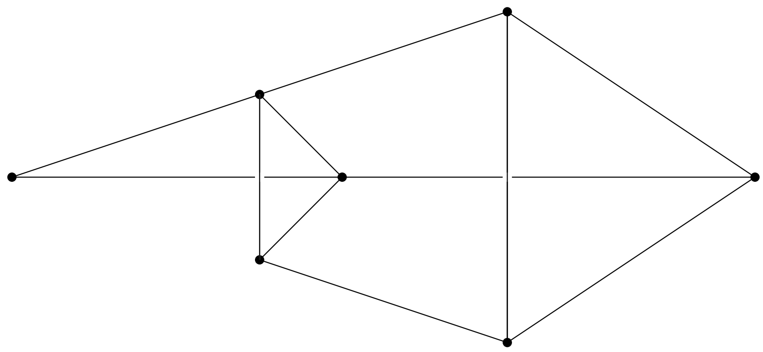
Matroid theory: exercise sheet 2

1. Prove that the following diagram does not depict a matroid:



2. Let M be a matroid on a set E. Let X be a subset of E which is both a circuit and a hyperplane. Prove that $\mathcal{B}(M) \cup X$ is the set of bases of a matroid on E.

3. Find all graphs G for which $M(G)$ has a set X as in exercise 2.

4.* Let M be a matroid. Prove that M is uniform if and only if every circuit of M meets every cocircuit of M.

5.* Let E be a finite set and let $\mathrm{Cl}\colon \mathcal{P}E \to \mathcal{P}E$ be a function with the following properties:

(CL1) For any $X \subseteq E$ we have $X \subseteq \mathrm{Cl}(X)$

(CL2) For any $X \subseteq Y \subseteq E$ we have $\mathrm{Cl}(X) \subseteq \mathrm{Cl}(Y)$

(CL3) For any $X \subseteq E$ we have $\mathrm{Cl}(\mathrm{Cl}(X)) = \mathrm{Cl}(X)$

(CL4) For any $X \subseteq E$, $x \in E$ und $y \in \mathrm{Cl}(X \cup x) - \mathrm{Cl}(X)$ we have $x \in \mathrm{Cl}(X \cup y)$

Prove that Cl is the closure operator of a matroid on E.