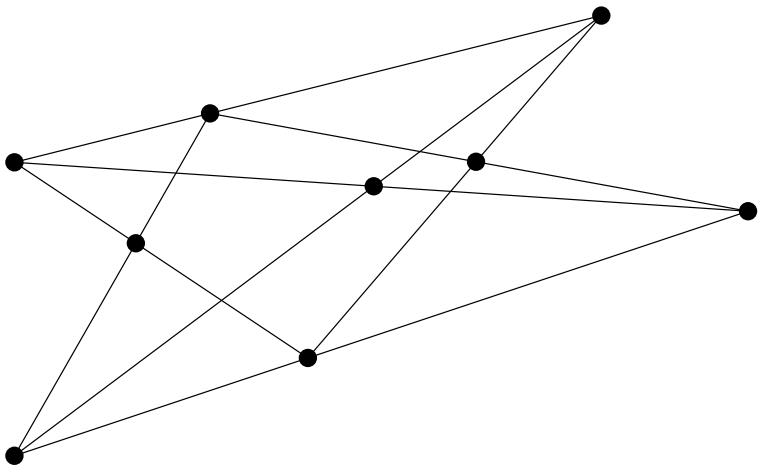
Matroid theory: exercise sheet 10

1. Prove that the matroid with the following geometric presentation is not representable over any field:



2. Let V be a subspace of k^E of dimension r, and let $\varphi\colon V^r \to k$ be an antisymmetric multilinear map with $\varphi \neq 0$. For $x_1 \dots x_r \in E$, let $\varphi_{x_1,\dots x_r}$ be the antisymmetric multilinear map sending $v_1 \dots v_r$ to $\det(v_i(x_j)|i,j \leq r)$. Let $\lambda(x_1,\dots x_r)$ be the unique element of k with $\varphi_{x_1\dots x_r} = \lambda(x_1,\dots x_r)\varphi$. Prove that λ is a Grassmann-Plücker function.

3. Let k be the finite field with q elements. Up to rescaling by $\mu \in (k^\times)^{\{1,2,3,4\}}$, how many subspaces V of $k^{\{1,2,3,4\}}$ are there with $M(V) = U_{2,4}$?

4* Show that for any s there is some n such that in any connected matroid with at least n elements there is a subset X of the ground set of size s with $M\backslash X$ or M/X connected.