Infinite matroid theory exercise sheet 9

1. (a) Prove that a multigraph G is 2-connected if and only if its finite cycle matroid is connected.

   (b) Prove that a multigraph G is 3-connected if and only if it is simple and its finite cycle matroid is 3-connected.

2. Let (M, N) be a twinned pair of matroids.

   (a) Show that any shift (with respect to M) of any normal base is again a normal base.

   (b) Show that for any partition $E = P \dot\cup Q$ of the common ground set $\kappa_M(P) = \kappa_N(P)$.

3. For two matroids M_1 and M_2 that share only one edge e, the *2-sum* $M_1 \oplus_2 M_2$ of M_1 and M_2 is the matroid whose edge set is the symmetric difference of those of M_1 and M_2 and whose scrawls are those symmetric differences of scrawls of M_1 and M_2 that do not contain the edge e.

   (a) Prove that $M_1 \oplus_2 M_2$ is a matroid.

   (b) Describe the bases, circuits and cocircuits of $M_1 \oplus_2 M_2$ in terms of the bases, circuits and cocircuits of M_1 and M_2, respectively. Prove that $M_1^* \oplus_2 M_2^* = (M_1 \oplus_2 M_2)^*$.

4.* Let M be a connected matroid such that every circuit of M and every cocircuit of M is countable. Prove that M is countable.

Hints

Concerning exercise 4: It might be helpful to think about fundamental circuits and cocircuits.