Infinite matroid theory exercise sheet 8

1. (a) Let G be a finitely separable graph and e an edge of G. Show that $|G - e|$ is homeomorphic to the space obtained from $|G|$ by removing all interior points of e.

   (b) A subspace of $|G|$ is a *standard subspace* if it is the closure of some set of edges of G. Show that any two vertices of a connected standard subspace of $|G|$ may be joined by an arc in that subspace.

2. (a) Let M be a connected matroid, and e be one of its edges. Prove that either $M/\{e\}$ or $M\backslash\{e\}$ is connected.

   (b)* Is it true for every $F \subseteq E(M)$ that there is a partition of F into sets A and B such that $M/A\backslash B$ is connected?

3.* Let M be a connected, finitary and cofinitary matroid. Prove that M is finite. Deduce that every matroid that is finitary and cofinitary is a direct sum of finite matroids.