Infinite matroid theory exercise sheet 6

1. Let G be a connected graph including no subdivision of the Bean Graph. Show that the algebraic cycle matroid of G is cofinitary if and only if G satisfies the following 2 conditions:

   - Every block of G is locally finite.
   - For any vertex v of G, only finitely many components of $G - v$ include a ray.

2. Let M be a matroid. Let $\mathcal{C}$ be the set of finite circuits of M. Show that $\mathcal{C}$ is the set of circuits of some matroid. This matroid is called the *finitarisation* M^{fin} of M. Is it true that if some base of M is a base of M^{fin}, then $M = M^{\text{fin}}$?

3. Let M be a finitary matroid, and N be a finite minor of M. Prove that there is a finite set C and some set D such that $N = M/C\backslash D$.

4.** Let M_f and M_c be a finitary matroid and a cofinitary matroid, respectively. Assume that $M_c^{\text{fin}} = M_f$, and that $(M_f^*)^{\text{fin}} = M_c^*$. Let M be a matroid such that $M^{\text{fin}} = M_f$, and that $(M^*)^{\text{fin}} = M_c^*$.

   Let F be some finite matroid. Is it true that if M_f and M_c do not have N as a minor, then M does not have N as a minor? Already for $F = U_{2,4}$ or $F = M(K_4)$ we do not know the answer.

Hints

Concerning question 4:

Suppose that $\mathcal{C}$ is the set of circuits of some matroid M. What do the minors of M look like?