Infinite matroid theory exercise sheet 5

1. Characterise for which graphs G, the algebraic cycle matroid $M_A(G)$ is cofinitary (that is, for which graphs G the scrawl system $\mathcal{D}_{AC}(G)$ is finitary).

2. Let G be a graph. Let $\mathcal{C}$ be the collection of edge sets of thetas, handcuffs, degenerate handcuffs, double rays and sperms in G, see Figure 1 and Figure 2. Prove that $\mathcal{C}$ is the set of circuits of some matroid (You may use exercise 3 from sheet 4).

3.* Let G be a graph that has no subdivision of the Bean graph. Show that $|C \cap D|$ is finite for every $C \in \overline{\mathcal{C}}_{AC}(G)$ and every $D \in \overline{\mathcal{D}}_{AC}(G)$ such that at least one rayless side of D is connected.

4.* Let G be a bipartite graph where all vertices of the left bipartition-class have finite degree. Show that there is a scrawl system whose ground set is the set of vertices on the left, and whose independent sets are precisely the sets of vertices on the left that can be matched into the right hand side.

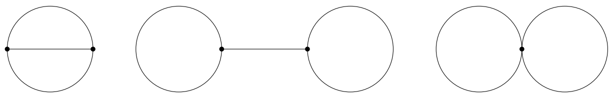

Figure 1: A *theta* is a subdivision of the graph on the left. A *handcuff* is a subdivision of the graph in the middle. A *degenerate handcuff* is a subdivision of the graph on the right.

Figure 2: A *double ray* is the graph on the left. A *sperm* is a subdivision of a the graph on the right.

Hints

In general: you may need to use Zorn's lemma and compactness arguments.
Concerning question 2: Suppose that $\mathcal{C}$ is the set of circuits of some matroid M. What do the minors of M look like?
Concerning question 3: Consider the proof of (O2) for $\overline{\mathcal{C}}_{AC}(G)$ and $\overline{\mathcal{D}}_{AC}(G)$.