# Infinite matroid theory exercise sheet 4

1. Show that if $\mathcal{C}$ satisfies infinite circuit elimination $(C3)$, then so does $\mathcal{C}^*$.

2. Let $X$ be some uncountable set, and let $y \notin X$. Let $\mathcal{I}$ consist of those subsets of $X + y$ that either are countable or do not contain $y$. Show that $\mathcal{I}$ satisfies (I1)-(I3) but there is no scrawl system whose set of independent sets is $\mathcal{I}$.

3. Let $G$ be a graph. Let $\mathcal{C}$ be the collection of edge sets of thetas, handcuffs, degenerate handcuffs, double rays and sperms in $G$, see Figure 1 and Figure 2. For every tree $T$, let $\partial T$ consist of those edges not in $T$ that have at least one endvertex in $T$. Let $\mathcal{D}$ consist of all sets $\partial T$ for (not necessary spanning) rayless trees $T$ in $G$. Show that the pair $(\mathcal{C}, \mathcal{D})$ satisfies (01) and (02). Deduce that $\mathcal{C}$ satisfies $(C3)$.

4*. Let $M$ be a matroid with two bases $B_1$ and $B_2$. Prove that there is a bijection $\alpha : B_1 \to B_2$ such that $B_1 - x + \alpha(x)$ is a base of $M$.

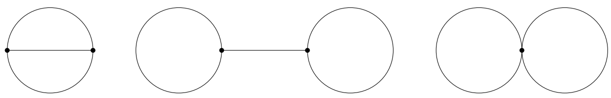

Figure 1: A *theta* is a subdivision of the graph on the left. A *handcuff* is a subdivision of the graph in the middle. A *degenerate handcuff* is a subdivision of the graph on the right.

Figure 2: A *double ray* is the graph on the left. A *sperm* is a subdivision of a the graph on the right.

## Hints

Concerning question 4:

Use Hall's marriage theorem.