Infinite matroid theory exercise sheet 12

1. Is there a finite field k such that every finite uniform matroid is representable over k?

2. (a) Let M be a tame matroid. Show that M is a binary thin sums matroid if and only if for any base B of M and any circuit C of M we have $C = \triangle_{e \in C \setminus B} C_e^B$.

   (b)** Let M be a tame matroid with some base B such that for any circuit C of M we have $C = \triangle_{e \in C \setminus B} C_e^B$. Must M be a binary thin sums matroid?

3. (a) Let $\mathcal{I}$ be the set of sets of edges of the Bean graph including no algebraic circuit (that is, no edge set of a finite cycle or a double ray). Show that $\mathcal{I}$ satisfies (IM) but not (I3).

   (b) Let $E = \mathbb{N} \times \{0, 1\}$ and define the function $\phi \colon E \to \mathbb{Q}^{\mathbb{N}}$ by $f((n, 0))(i) = i^n$ and $f((n, 1))(i) = -1$ if $n = i$ and 0 otherwise. Show that the set $\mathbb{N} \times \{0\}$ is thinly independent with respect to ϕ. Show that the set of thinly independent sets with respect to ϕ doesn't satisfy (IM) by showing that it doesn't include a maximal element extending $\mathbb{N} \times \{0\}$.

4.* Show that a matroid M is linearly representable by a family of vectors in some vector space over k if and it is both finitary and thin sums representable over k.

5.** Is every binary thin sums matroid tame?