# Infinite matroid theory exercise sheet 10

1. (a) Let P be a subset of the ground set of a matroid M and let P_0 be any connected component of $M{\restriction}_P$. Prove that $\kappa_M(P_0) \le \kappa_M(P)$.

   (b) Prove that the finite cycle matroid of any 3-connected graph is 3-connected.

2. Let (M, N) be a twinned pair of matroids. By considering shifting sequences of normal bases, show that for any partition $E = P \dot\cup Q$ of the common ground set $\kappa_M(P) = \kappa_N(P)$.

3. We say that two topological circuits of the graph pictured below are *equivalent* if their symmetric difference is finite. Let $\mathcal{C}$ be any union of equivalence classes which includes the class of all finite circuits.

   (a) Prove that $\mathcal{C}$ is the set of circuits of a matroid.

   (b) What does the canonical tree decomposition of this matroid over its 2-separations look like? What are the torsos?
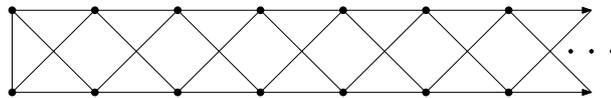
4.* Let M be a connected matroid such that every circuit of M and every cocircuit of M is countable. Prove that M is countable.