UNIVERSITEIT VAN AMSTERDAM
INSTITUTE FOR LOGIC, LANGUAGE AND COMPUTATION

Core Logic

2004/2005; 1st Semester
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Homework Set # 8 *Deadline:* November 10th, 2004

Exercise 22 (3 points total).

Give the names of the following people (1 point each):

- X was a Aristotelian philosopher from Constantinople who lived in Italy most of his life. From 1456 to 1458, he was the professor for rhetoric and poetics at the *studio fiorentino* and one of the teachers of Lorenzo de'Medici (*il Magnifico*).
- Y was one of the authors of *La logique, ou l'art de penser*. He was called "the Great" to distinguish him from his father who had the same name.
- Z was a niece of King Frederick the Great of Prussia. She was the recipient of the *Lettres à une Princesse d'Allemande* in which Euler explained deductive reasoning by what we now call "Euler diagrams".

Exercise 23 (10 points).

A structure $\langle R, +, \cdot, 0, 1\rangle$ is called a **ring** if $+$ is commutative and associative binary operation on R, $\cdot$ is an associative binary operation on R, $\cdot$ distributes over $+$ (*i.e.*, $a\cdot(b+c) = a\cdot b+a\cdot c$ and $(a+b)\cdot c = a\cdot c+b\cdot c$), 0 is the neutral element of $+$ (*i.e.*, $0+a = a+0 = a$) and 1 is the neutral element of $\cdot$ (*i.e.*, $a\cdot 1 = 1\cdot a = a$).
Examples of rings are: the integers $\mathbb{Z}$, the rationals $\mathbb{Q}$, the reals $\mathbb{R}$.
Let $\mathbf{B} = \langle B, 0, 1, \wedge, \vee, -\rangle$ be a Boolean algebra. For $X, Y \in B$, define

$$X + Y := (X \wedge -Y) \vee (-X \wedge Y), \text{ and}$$
$$X \cdot Y := X \wedge Y.$$

We write $\mathrm{R}(\mathbf{B}) := \langle B, +, \cdot, 0, 1\rangle$.

(1) Prove that $\mathrm{R}(\mathbf{B})$ is a ring (6 points).
(2) Give an example of a ring R such that R is not isomorphic to any $\mathrm{R}(\mathbf{B})$ (with a proof; 4 points).

Exercise 24 (6 points total).

Let $\mathcal{L} = \{P\}$ be the language with one binary relation symbol. Consider the following $\mathcal{L}$-sentence σ:

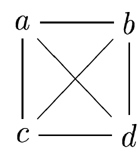
$$\forall x\,(\exists y\,(\exists z\,(P(x,y) \wedge P(z,y) \wedge (P(x,z) \to P(z,x))))).$$

For each of the following three models $\mathbf{M}_0$, $\mathbf{M}_1$, and $\mathbf{M}_2$, determine whether $\mathbf{M}_i \models \sigma$ or not. Give a brief argument (2 points each).

(1) $\mathbf{M}_0 := \langle \mathbb{N}, R_0 \rangle$ where nR_0m if and only if $n < m$.
(2) $\mathbf{M}_1 := \langle \mathbb{N}, R_1 \rangle$ where nR_1m if and only if $m = 2n$.
(3) $\mathbf{M}_2 := \langle \mathbb{N}, R_2 \rangle$ where nR_2m if and only if $n > m$.

Exercise 25 (6 points plus 3 extra points).

Consider the following plane geometry:



Let $P = \{a, b, c, d\}$, $L = \{\{a, b\}, \{a, c\}, \{a, d\}, \{b, c\}, \{b, d\}, \{c, d\}\}$ and $pI\ell$ if $p \in \ell$. Show that $\mathbf{P} := \langle P, L, I \rangle$ is a strongly Euclidean plane. (Note that the picture is potentially misleading: there is no intersection of the two diagonal lines in $\mathbf{P}$; 6 points.)

For students with a mathematical background: What does this example have to do with the two-dimensional vector space over the field $\mathbb{Z}/(2)$? (1 extra point). Can you come up with an analogous example for the two-dimensional vector space over the field $\mathbb{Z}/(3)$? (2 extra points)