Core Logic
2004/2005; 1st Semester
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Homework Set # 13   *Deadline:* December 15th, 2004

Exercise 41 (10 points total).

In his guest lecture, Johan van Benthem introduced a Kripke model solution to the muddy children puzzle. (Check his "One is a lonely number" on the course webpage.) In his version, child 1 and 2 were dirty and child 3 was clean (we write **DDC** for this situation). The father announced that one of them was dirty, eliminating the node **CCC**. The children were allowed to state "I don't know my status", "I am dirty" and "I am clean". In each round, all children simultaneously had to state their epistemic status truthfully.

Let us change the setting slightly: Instead of stating their epistemic status simultaneously, they do it successively. In the first round, child 1 states his status, in the second child 2, then in the third round, child 3, and so on.

 (1) Show (using the Kripke model formalism) that in this scenario with the true status **DDC**, the second child announces his status in round 2, the third child announces his status in round 3, and child 1 will never find out whether he's dirty or clean (5 points).
 (2) Do the same analysis for the true situation **CDD**. What changes? (5 points)

Exercise 42 (7 points total).

What is an Erdős number? This can either be a large cardinal notion (give a one-sentence description; 1 point) or a property of researchers (give a full recursive definition of "X has Erdős number n"; 4 points).

Why do most set theorists have Erdős number ≤ 4? (2 points)

Hint. It has to do with a very prolific author who is working in set theory and has low Erdős number.

Exercise 43 (8 points total).

Let $\langle \mathbf{M}, V \rangle$ be a Kripke model. We define

$$\mathbf{not}\, \varphi := \Box \neg \varphi.$$

Let DN_0 (for "*duplex negatio*") be $\mathbf{not}\,\mathbf{not}\,\varphi \to \varphi$ and DN_1 be $\varphi \to \mathbf{not}\,\mathbf{not}\,\varphi$.

 (1) Do DN_0 and DN_1 hold in the class of all reflexive, transitive frames ("**S4**-frames"; 2 points each)?
 (2) Do DN_0 and DN_1 hold in the class of all reflexive, symmetric, transitive frames ("**S5**-frames"; 2 points each)? *[What does this have to do with **S5***? (1 extra point)]*