UNIVERSITEIT VAN AMSTERDAM
INSTITUTE FOR LOGIC, LANGUAGE AND COMPUTATION

# Advanced Topics in Set Theory

**2004/2005; 1st Semester**
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**Homework Set # 10** *Deadline:* December 9th, 2004

**Exercise A**.
Finish the proof of the theorem that there are no non-principal ultrafilters on $\omega$ under $\mathsf{AD}$. More specifically, look at the game $\mathrm{G}(U)$ where players I and II play mutually disjoint finite sets $X_i \subseteq \mathbb{N}$, and player I wins if $\bigcup_{i\in\omega} X_{2i} \in U$. Show that if $U$ is a nonprincipal ultrafilter and $\tau$ is a winning strategy for player II in $\mathrm{G}(U)$, then you can define a strategy $\sigma_\tau$ for player I which is a winning strategy. (Of course, this is a contradiction.)

**Exercise B** (8.43 from Andretta).
Show under $\mathsf{AD}$ that any $f : \mathcal{D} \to \mathbb{R}$ is constant on a cone (*i.e.*, there is a cone $C \subseteq \mathcal{D}$ such that $f[C]$ is a singleton).
Use this to show that there is no choice function for the Turing degrees.

**Exercise C** (8.71 from Andretta).
Read Andretta's definition of $\mathsf{AD}^{\mathbb{R}}_{½}$. Define $\mathsf{AD}^{\wp(\mathbb{R})}_{½}$ and $\mathsf{AD}^{\omega_1}_{½}$ analogously and prove that they are inconsistent.