Universiteit van Amsterdam
Institute for Logic, Language and Computation

Recursion Theory

2003/2004; 1st Semester
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Homework Set # 10. Deadline: December 4th, 2003

Exercise # 1.
Let $f : \mathbb{N} \to \mathbb{N}$ be a function. Define a sequence of sets of natural numbers as follows:

- $E_{-1} := \varnothing$,
- $E_n := \mathrm{W}_{f(n)}^{E_{n-1}}$.

Show that there is a set $H \subseteq \mathbb{N}$ such that for all $n \in \mathbb{N}$, $E_n <_{\mathrm{T}} H$. Deduce from this that there is a set H such that for all $n \in \mathbb{N}$, $\mathbf{0}^{(n)} <_{\mathrm{T}} H$.

Exercise # 2.
A subset $C \subseteq \mathcal{D}$ is called a **cone** if there is some $\mathbf{b} \in \mathcal{D}$ (called the **base of the cone**) such that

$$C = \{\mathbf{d} \in \mathcal{D}\,;\, \mathbf{b} \leq_{\mathrm{T}} \mathbf{d}\}.$$

Define the family $\mathrm{MF} \subseteq \wp(\mathcal{D})$ by

$$X \in \mathrm{MF} :\Longleftrightarrow \quad \text{there is a cone } C \text{ such that } C \subseteq X.$$

This family is called the **Martin filter**. Show that it is actually a filter, *i.e.*, (i) $\mathcal{D} \in \mathrm{MF}$, (ii) supersets of sets in MF are in MF, and (iii) intersections of sets in MF are in MF.